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Research on Some Fractional Differential Problems Based on Jumarie Type of Riemann-Liouville Fractional Derivative

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Abstract: **In this paper, based on Jumarie's modified Riemann-Liouville (R-L) fractional derivative and a new multiplication of fractional analytic functions, we obtain arbitrary order fractional derivative of two types of matrix fractional functions by using some methods. In fact, our results are generalizations of traditional calculus results.**

Keywords: **Jumarie's modified R-L fractional derivative, new multiplication, fractional analytic functions, matrix fractional functions.**

I. INTRODUCTION

Fractional calculus is a natural extension of the traditional calculus. In fact, since the beginning of the theory of differential and integral calculus, some mathematicians have studied their ideas on the calculation of non-integer order derivatives and integrals. During the 18th and 19th centuries, there were many famous scientists such as Euler, Laplace, Fourier, Abel, Liouville, Grunwald, Letnikov, Riemann, Laurent, Heaviside, and some others who reported interesting results within fractional calculus. In recent years, fractional calculus has become an increasingly popular research area due to its effective applications in different scientific fields such as economics, viscoelasticity, physics, mechanics, biology, electrical engineering, control theory, and so on [1-12].

However, different from the traditional calculus, the rule of fractional derivative is not unique, many scholars have given the definitions of fractional derivatives. The common definition is Riemann-Liouville (R-L) fractional derivatives. Other useful definitions include Caputo fractional derivatives, Grunwald-Letnikov (G-L) fractional derivatives, and Jumarie type of R-L fractional derivatives to avoid non-zero fractional derivative of constant function [13-17].

In this paper, based on Jumarie type of Riemann-Liouville (R-L) fractional derivative and a new multiplication of fractional analytic functions, we obtain arbitrary order fractional derivative of the following two types of matrix fractional functions:

$$\left(cos_\alpha\left(E_\alpha\left(tA\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)\right)\right)^{\otimes_\alpha p},$$

and

$$\left(sin_\alpha\left(E_\alpha\left(tA\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)\right)\right)^{\otimes_\alpha p},$$

where $0<\alpha\leq 1$, p is a positive integer, t is a real number, and A is a real matrix. In addition, our results are generalizations of classical calculus results.

II. PRELIMINARIES

At first, we introduce the fractional derivative used in this paper.

Definition 2.1 ([18])**:** Let $0<\alpha\leq 1$, and x_0 be a real number. The Jumarie's modified Riemann-Liouville (R-L) α-fractional derivative is defined by

$$\left({}_{x_0}D_x^{\alpha}\right)[f(x)]=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{x_0}^{x}\frac{f(t)-f(x_0)}{(x-t)^{\alpha}}dt\ , \tag{1}$$

where $\Gamma(\ \ )$ is the gamma function. On the other hand, for any positive integer m, we define $\left({}_{x_0}D_x^{\alpha}\right)^m[f(x)]=\left({}_{x_0}D_x^{\alpha}\right)\left({}_{x_0}D_x^{\alpha}\right)\cdots\left({}_{x_0}D_x^{\alpha}\right)[f(x)]$, the m-th order α-fractional derivative of $f(x)$.

Proposition 2.2 ([19])**:** *If α,β,x_0,C are real numbers and $\beta\geq\alpha>0$, then*

$$\left({}_{x_0}D_x^{\alpha}\right)\left[(x-x_0)^{\beta}\right]=\frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(x-x_0)^{\beta-\alpha}, \tag{2}$$

and

$$\left({}_{x_0}D_x^{\alpha}\right)[C]=0 \quad .$$

(3)

Definition 2.3 ([20])**:** If x,x_0, and a_n are real numbers for all n, $x_0\in(a,b)$, and $0<\alpha\leq 1$. If the function $f_{\alpha}:[a,b]\to R$ can be expressed as an α-fractional power series, that is, $f_{\alpha}(x^{\alpha})=\sum_{n=0}^{\infty}\frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}$ on some open interval containing x_0, then we say that $f_{\alpha}(x^{\alpha})$ is α-fractional analytic at x_0. Furthermore, if $f_{\alpha}:[a,b]\to R$ is continuous on closed interval $[a,b]$ and it is α-fractional analytic at every point in open interval (a,b), then f_{α} is called an α-fractional analytic function on $[a,b]$.

In the following, we introduce a new multiplication of fractional analytic functions.

Definition 2.4 ([21])**:** If $0<\alpha\leq 1$. Assume that $f_{\alpha}(x^{\alpha})$ and $g_{\alpha}(x^{\alpha})$ are two α-fractional power series at $x=x_0$,

$$f_{\alpha}(x^{\alpha})=\sum_{n=0}^{\infty}\frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}, \tag{4}$$

$$g_{\alpha}(x^{\alpha})=\sum_{n=0}^{\infty}\frac{b_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}. \tag{5}$$

Then

$$\begin{aligned}&f_{\alpha}(x^{\alpha})\otimes_{\alpha}g_{\alpha}(x^{\alpha})\\&=\sum_{n=0}^{\infty}\frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}\otimes_{\alpha}\sum_{n=0}^{\infty}\frac{b_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}\\&=\sum_{n=0}^{\infty}\frac{1}{\Gamma(n\alpha+1)}\left(\sum_{m=0}^{n}\binom{n}{m}a_{n-m}b_m\right)(x-x_0)^{n\alpha}.\end{aligned} \tag{6}$$

Equivalently,

$$\begin{aligned}&f_{\alpha}(x^{\alpha})\otimes_{\alpha}g_{\alpha}(x^{\alpha})\\&=\sum_{n=0}^{\infty}\frac{a_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes_{\alpha}n}\otimes_{\alpha}\sum_{n=0}^{\infty}\frac{b_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes_{\alpha}n}\\&=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\sum_{m=0}^{n}\binom{n}{m}a_{n-m}b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes_{\alpha}n}.\end{aligned} \tag{7}$$

Definition 2.5 ([22])**:** If $0<\alpha\leq 1$, and x is a real number. The α-fractional exponential function is defined by

$$E_{\alpha}(x^{\alpha})=\sum_{n=0}^{\infty}\frac{x^{n\alpha}}{\Gamma(n\alpha+1)}=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_{\alpha}n}. \tag{8}$$

On the other hand, the α-fractional cosine and sine function are defined as follows:

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$$cos_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n\alpha}}{\Gamma(2n\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 2n}, \tag{9}$$

and

$$sin_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha (2n+1)}. \tag{10}$$

Definition 2.6 ([23])**:** If $0 < \alpha \leq 1$, and A is a matrix. The matrix α-fractional exponential function is defined by

$$E_\alpha(Ax^\alpha) = \sum_{n=0}^{\infty} A^n \frac{x^{n\alpha}}{\Gamma(n\alpha+1)} = \sum_{n=0}^{\infty} \frac{1}{n!}\left(A\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha n}. \tag{11}$$

And the matrix α-fractional cosine and sine function are defined as follows:

$$cos_\alpha(Ax^\alpha) = \sum_{n=0}^{\infty} A^n \frac{(-1)^n x^{2n\alpha}}{\Gamma(2n\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(A\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 2n}, \tag{12}$$

and

$$sin_\alpha(Ax^\alpha) = \sum_{n=0}^{\infty} A^n \frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(A\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha (2n+1)}. \tag{13}$$

Theorem 2.7 (matrix fractional Euler's formula)([24])**:** *If* $0 < \alpha \leq 1$, $i = \sqrt{-1}$, *and A is a real matrix, then*

$$E_\alpha(iAx^\alpha) = cos_\alpha(Ax^\alpha) + i sin_\alpha(Ax^\alpha). \tag{14}$$

Theorem 2.8 (matrix fractional DeMoivre's formula)([24])**:** *If* $0 < \alpha \leq 1$, *p is an integer, and A is a real matrix, then*

$$[cos_\alpha(Ax^\alpha) + i sin_\alpha(Ax^\alpha)]^{\otimes_\alpha p} = cos_\alpha(pAx^\alpha) + i sin_\alpha(pAx^\alpha). \tag{15}$$

Theorem 2.9 (fractional binomial theorem)([25])**:** *If* $0 < \alpha \leq 1$, *p is a positive integer and* $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ *are two* α-*fractional analytic functions. Then*

$$[f_\alpha(x^\alpha) + g_\alpha(x^\alpha)]^{\otimes_\alpha p} = \sum_{k=0}^{p} \binom{p}{k} \left(f_\alpha(x^\alpha)\right)^{\otimes_\alpha (p-k)} \otimes_\alpha \left(g_\alpha(x^\alpha)\right)^{\otimes_\alpha k}, \tag{16}$$

where $\binom{p}{k} = \frac{p!}{k!(p-k)!}$.

III. MAIN RESULTS

In this section, we find arbitrary order fractional derivative of two types of matrix fractional functions. At first, a lemma is needed.

Lemma 3.1: *If* $0 < \alpha \leq 1$, *t is a real number, p is a positive integer, and A is a real matrix, then*

$$\left(cos_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p} = \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}E_\alpha(2ntAx^\alpha), \tag{17}$$

and

$$\left(sin_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p} = \frac{1}{(-2)^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\begin{bmatrix} cos\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}E_\alpha(2ntAx^\alpha) \\ -sin\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(p-2k)^{2n+1}E_\alpha((2n+1)tAx^\alpha)\end{bmatrix}. \tag{18}$$

Proof: $\left(cos_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p}$

$$= \left(\frac{1}{2}\left[E_\alpha\left(iE_\alpha(tAx^\alpha)\right) + E_\alpha\left(-iE_\alpha(tAx^\alpha)\right)\right]\right)^{\otimes_\alpha p}$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\left[E_\alpha\left(iE_\alpha(tAx^\alpha)\right)\right]^{\otimes_\alpha (p-k)} \otimes_\alpha \left[E_\alpha\left(-iE_\alpha(tAx^\alpha)\right)\right]^{\otimes_\alpha k} \quad \text{(by fractional binomial theorem)}$$

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$$= \frac{1}{2^p} \sum_{k=0}^{p} \binom{p}{k} E_\alpha\left(i(p-k)E_\alpha(tAx^\alpha)\right) \otimes_\alpha E_\alpha\left(-ikE_\alpha(tAx^\alpha)\right) \quad \text{(by matrix fractional DeMoivre's formula)}$$

$$= \frac{1}{2^p} \sum_{k=0}^{p} \binom{p}{k} E_\alpha\left(i(p-2k)E_\alpha(tAx^\alpha)\right)$$

$$= \frac{1}{2^p} \sum_{k=0}^{p} \binom{p}{k} cos_\alpha\left((p-2k)E_\alpha(tAx^\alpha)\right)$$

$$= \frac{1}{2^p} \sum_{k=0}^{p} \binom{p}{k} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left((p-2k)E_\alpha(tAx^\alpha)\right)^{\otimes_\alpha 2n}$$

$$= \frac{1}{2^p} \sum_{k=0}^{p} \binom{p}{k} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (p-2k)^{2n} E_\alpha(2ntAx^\alpha).$$

And

$$\left(sin_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p}$$

$$= \left(\frac{1}{2i}\left[E_\alpha\left(iE_\alpha(tAx^\alpha)\right) - E_\alpha\left(-iE_\alpha(tAx^\alpha)\right)\right]\right)^{\otimes_\alpha p}$$

$$= \frac{i^p}{(-2)^p} \sum_{k=0}^{p} \binom{p}{k} \left[E_\alpha\left(iE_\alpha(tAx^\alpha)\right)\right]^{\otimes_\alpha (p-k)} \otimes_\alpha \left[-E_\alpha\left(-iE_\alpha(tAx^\alpha)\right)\right]^{\otimes_\alpha k} \quad \text{(by fractional binomial theorem)}$$

$$= \frac{i^p}{(-2)^p} \sum_{k=0}^{p} \binom{p}{k} (-1)^k E_\alpha\left(i(p-k)E_\alpha(tAx^\alpha)\right) \otimes_\alpha E_\alpha\left(-ikE_\alpha(tAx^\alpha)\right) \quad \text{(by matrix fractional DeMoivre's formula)}$$

$$= \frac{1}{(-2)^p} \left[cos\left(\frac{p\pi}{2}\right) + isin\left(\frac{p\pi}{2}\right)\right] \sum_{k=0}^{p} \binom{p}{k} (-1)^k E_\alpha\left(i(p-2k)E_\alpha(tAx^\alpha)\right)$$

$$= \frac{1}{(-2)^p} \left[cos\left(\frac{p\pi}{2}\right) + isin\left(\frac{p\pi}{2}\right)\right] \sum_{k=0}^{p} \binom{p}{k} (-1)^k \left[cos_\alpha\left((p-2k)E_\alpha(tAx^\alpha)\right) + isin_\alpha\left((p-2k)E_\alpha(tAx^\alpha)\right)\right]$$

$$= \frac{1}{(-2)^p} \sum_{k=0}^{p} \binom{p}{k} (-1)^k \left[cos\left(\frac{p\pi}{2}\right) cos_\alpha\left((p-2k)E_\alpha(tAx^\alpha)\right) - sin\left(\frac{p\pi}{2}\right) sin_\alpha\left((p-2k)E_\alpha(tAx^\alpha)\right)\right]$$

$$= \frac{1}{(-2)^p} \sum_{k=0}^{p} \binom{p}{k} (-1)^k \left[\begin{array}{c} cos\left(\frac{p\pi}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left((p-2k)E_\alpha(tAx^\alpha)\right)^{\otimes_\alpha 2n} \\ -sin\left(\frac{p\pi}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left((p-2k)E_\alpha(tAx^\alpha)\right)^{\otimes_\alpha (2n+1)} \end{array}\right]$$

$$= \frac{1}{(-2)^p} \sum_{k=0}^{p} \binom{p}{k} (-1)^k \left[\begin{array}{c} cos\left(\frac{p\pi}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (p-2k)^{2n} E_\alpha(2ntAx^\alpha) \\ -sin\left(\frac{p\pi}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (p-2k)^{2n+1} E_\alpha((2n+1)tAx^\alpha) \end{array}\right]. \quad \text{q.e.d.}$$

Theorem 3.2: *If* $0 < \alpha \leq 1$, t *is a real number,* m, p *are positive integers, and* A *is a real matrix, then*

$$\left({}_0D_x^\alpha\right)^m \left[\left(cos_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p}\right] = \frac{1}{2^p} (tA)^m \sum_{k=0}^{p} \binom{p}{k} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (p-2k)^{2n} (2n)^m E_\alpha(2ntAx^\alpha). \quad (19)$$

And

$$\left({}_0D_x^\alpha\right)^m \left[\left(sin_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p}\right]$$

$$= \frac{1}{(-2)^p} (tA)^m \sum_{k=0}^{p} \binom{p}{k} (-1)^k \left[\begin{array}{c} cos\left(\frac{p\pi}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (p-2k)^{2n} (2n)^m E_\alpha(2ntAx^\alpha) \\ -sin\left(\frac{p\pi}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (p-2k)^{2n+1} (2n+1)^m E_\alpha((2n+1)tAx^\alpha) \end{array}\right]. \quad (20)$$

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Proof: By Lemma 3.1, we have

$$\left({}_0D_x^\alpha\right)^m\left[\left(cos_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p}\right]$$

$$=\left({}_0D_x^\alpha\right)^m\left[\frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}E_\alpha(2ntAx^\alpha)\right]$$

$$=\frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}\left({}_0D_x^\alpha\right)^m[E_\alpha(2ntAx^\alpha)]$$

$$=\frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}(2n)^m(tA)^mE_\alpha(2ntAx^\alpha)$$

$$=\frac{1}{2^p}(tA)^m\sum_{k=0}^{p}\binom{p}{k}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}(2n)^mE_\alpha(2ntAx^\alpha)\,.$$

On the other hand,

$$\left({}_0D_x^\alpha\right)^m\left[\left(sin_\alpha\left(E_\alpha(tAx^\alpha)\right)\right)^{\otimes_\alpha p}\right]$$

$$=\left({}_0D_x^\alpha\right)^m\left[\frac{1}{(-2)^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\left[\begin{array}{c}cos\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}E_\alpha(2ntAx^\alpha)\\-sin\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(p-2k)^{2n+1}E_\alpha((2n+1)tAx^\alpha)\end{array}\right]\right]$$

$$=\frac{1}{(-2)^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\left[\begin{array}{c}cos\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}\left({}_0D_x^\alpha\right)^m[E_\alpha(2ntAx^\alpha)]\\-sin\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(p-2k)^{2n+1}\left({}_0D_x^\alpha\right)^m[E_\alpha((2n+1)tAx^\alpha)]\end{array}\right]$$

$$=\frac{1}{(-2)^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\left[\begin{array}{c}cos\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}(2n)^m(tA)^mE_\alpha(2ntAx^\alpha)\\-sin\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(p-2k)^{2n+1}(2n+1)^m(tA)^mE_\alpha((2n+1)tAx^\alpha)\end{array}\right]$$

$$=\frac{1}{(-2)^p}(tA)^m\sum_{k=0}^{p}\binom{p}{k}(-1)^k\left[\begin{array}{c}cos\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(p-2k)^{2n}(2n)^mE_\alpha(2ntAx^\alpha)\\-sin\left(\frac{p\pi}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(p-2k)^{2n+1}(2n+1)^mE_\alpha((2n+1)tAx^\alpha)\end{array}\right].$$

q.e.d.

IV. CONCLUSION

In this paper, based on Jumarie's modified R-L fractional derivative and a new multiplication of fractional analytic functions, we obtain arbitrary order fractional derivative of two types of matrix fractional functions by using some methods. Moreover, our results are generalizations of ordinary calculus results. In the future, we will continue to use Jumarie type of R-L fractional calculus and the new multiplication of fractional analytic functions to solve problems in engineering mathematics and fractional differential equations.

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